

Proiect de lecție

Date Generale:

Profesor : Cercel Mirela Livia

Scoala : Colegiul National « Nichita Stanescu »

Disciplina : Matematica

Clasa : a-VIII-a

Capitolul : Numere reale

Tipul de lectie : Recapitularea cunostintelor dobandite de elevi :

- Sa stie sa compare si sa reprezinte pe axa numerele reale.
- Sa efectueze calcule cu numere reale, utilizand proprietatile operatiilor

Durata lectiei : 50 minute

Locul de desfasurare : sala de clasa

Data : 02 .11.2009

Elemente anterior invatate :

-Multimi de numere reale : $N \subset Z \subset Q \subset R$. Modulul unui numar real.

Intervale de numere reale . Operatii cu intervale . Operatii cu numere reale . Calcule cu radicali . Reguli de calcul .

Rationalizarea numitorilor de forma : $a\sqrt{b}$; $a \pm \sqrt{b}$ ($a, b \in N^*$)

Elemente noi de continut :

-efectuarea unor aplicatii care sa fixeze elementele anterior invatate si aplicarea lor in exercitii cu grad de dificultate mai mare

Obiective operationale :

La sfarsitul lectiei elevii vor fi capabili sa efectueze :

- operatii cu intervale de numere reale
- operatii cu numere reale , calcule cu radicali , rationalizari

Organizarea clasei pentru lectie :

Mijloace utilizate : -tabla
-caiet , culegere
-culegere de probleme

Strategii didactice: conversatia, explicatia, problematizarea, exercitiul

Continutul lectiei:

1. Stabiliti elementele multimilor:

$$A = \{x \in \mathbb{Z} \mid \frac{8}{x+2} \in \mathbb{Z}\}$$

$$B = \{x \in \mathbb{R} \mid |x+2| < 4\}$$

Sa se efectueze $A \cup B$, $A \cap B$, A/B si B/A .

Rezolvare :

$$\frac{8}{x+2} \in \mathbb{Z} \Rightarrow x+2 \mid 8$$

$$D_8 = \{\pm 1, \pm 2, \pm 4, \pm 8\}$$

$$x+2=1 \Rightarrow x=-1$$

$$x+2=-1 \Rightarrow x=-3$$

$$x+2=2 \Rightarrow x=0$$

$$x+2=-2 \Rightarrow x=-4$$

$$x+2=4 \Rightarrow x=2$$

$$x+2=-4 \Rightarrow x=-6$$

$$x+2=8 \Rightarrow x=6$$

$$x+2=-8 \Rightarrow x=-10$$

$$A = \{-10 ; -6 ; -4 ; -3 ; -1 ; 0 ; 2 ; 6\}$$

$$|x+2| < 4 \Rightarrow -4 < x+2 < 4$$

$$-4-2 < x < 2$$

$$-6 < x < 2$$

$$B = (-6, 2)$$

$$A = \{-10, -6, -4, -3, -1, 0, 2, 6\}$$

$$A \cup B = [-6, 2] \cup \{-10, 6\}$$

$$A \cap B = \{-4, -3, -1, 0\}$$

$$A \setminus B = \{-10, -6, 2, 6\}$$

$$B \setminus A = (-6, -4) \cup (-4, -3) \cup (-3, -1) \cup (-1, 0) \cup (0, 2)$$

$$\dots |^{-10} \dots (-^{-6} \dots |^{-4} \dots |^{-3} \dots |^{-1} \dots |^0 \dots |^1 \dots ^2) \dots |^6 \dots$$

2. Aratati ca $x = \sqrt{(a-2)^2} + \sqrt{(a+1)^2} \in \mathbb{N}$ pentru ca orice valoare reala a lui a pentru $-1 \leq a < 2$

Rezolvare :

$$x = \sqrt{(a-2)^2} + \sqrt{(a+1)^2} = |a-2| + |a+1|$$

$$-1 \leq a < 2$$

$$-1-2 \leq a-2 < 2-2$$

$$-3 \leq a-2 < 0 \Rightarrow |a-2| = -a+1$$

$$-1 \leq a < 2$$

$$-1+1 \leq a+1 < 2+1$$

$$0 \leq a+1 < 3 \Rightarrow |a+1| = a+1$$

$$x = -a+2+a+1 = 3 \in \mathbb{N}$$

3. Sa se calculeze :

$$\frac{\sqrt{2-1}}{\sqrt{2}} + \frac{\sqrt{3}-\sqrt{2}}{\sqrt{6}} + \frac{\sqrt{4}-\sqrt{3}}{\sqrt{12}} + \frac{\sqrt{5}-\sqrt{4}}{\sqrt{20}}$$

Rezolvare :

$$= \frac{2-\sqrt{2}}{2} + \frac{\sqrt{18}-\sqrt{12}}{6} + \frac{\sqrt{12}-3}{6} + \frac{5-\sqrt{20}}{10}$$

$$\begin{aligned}
&= \frac{15}{2} \frac{2-\sqrt{2}}{2} + \frac{5}{6} \frac{3\sqrt{2}-2\sqrt{3}}{6} + \frac{5}{6} \frac{2\sqrt{3}-3}{6} + \frac{3}{10} \frac{5-2\sqrt{5}}{10} \\
&= \frac{30-15\sqrt{2}+15\sqrt{2}-10\sqrt{3}+10\sqrt{3}-15+15-6\sqrt{5}}{30} \\
&= \frac{30-6\sqrt{5}}{30} = \frac{6(5-\sqrt{5})}{30} = \frac{5-\sqrt{5}}{5}
\end{aligned}$$

4. Sa se calculeze media aritmetica ,geometrica si media armonica a numerelor:

$$\begin{aligned}
X &= (2+\sqrt{5})(2-\sqrt{5})^2 \\
y &= \sqrt{(\sqrt{5}-3)^2} + 2\sqrt{(1-\sqrt{5})^2} + \sqrt{(-1)^2}
\end{aligned}$$

Rezolvare:

$$\begin{aligned}
x &= (2+\sqrt{5})(2-\sqrt{5})(2-\sqrt{5}) = (4-5)(2-\sqrt{5}) \\
x &= -1(2-\sqrt{5}) = -2+\sqrt{5} \\
y &= |\sqrt{5}-3| + 2|1-\sqrt{5}| + 1 = \quad - \\
&= -\sqrt{5}+3+2(-1+\sqrt{5})+1 = -\sqrt{5}+3-2+2\sqrt{5}+1
\end{aligned}$$

$$y = -\sqrt{5}+1+2\sqrt{5}+1 = 2+\sqrt{5}$$

$$x = -2+\sqrt{5}$$

$$y = 2+\sqrt{5}$$

$$ma = \frac{x+y}{2} = \frac{-2+\sqrt{5}+2+\sqrt{5}}{2} = \frac{2\sqrt{5}}{2} = \sqrt{5}$$

$$mg = \sqrt{(-2+\sqrt{5})(2+\sqrt{5})} = \sqrt{\sqrt{5}^2 - 2^2} = \sqrt{5-4} = \sqrt{1} = 1$$

$$mh = \frac{2}{\frac{1}{x} + \frac{1}{y}} = \frac{2}{\frac{1}{-2+\sqrt{5}} + \frac{1}{2+\sqrt{5}}}$$

$$mh = \frac{2}{\frac{-2-\sqrt{5}}{4-5} + \frac{2-\sqrt{5}}{4-5}} = \frac{2}{\frac{-2-\sqrt{5}+2-\sqrt{5}}{-1}} = \frac{2}{\frac{-2\sqrt{5}}{-1}} = \frac{2}{2\sqrt{5}} = \frac{1}{\sqrt{5}}$$

5. Sa se compare numerele:

$$a = \sqrt{\frac{4-\sqrt{7}}{4+\sqrt{7}}} + \sqrt{\frac{\sqrt{7}-2}{3(\sqrt{7}+2)}}$$

$$b = \sqrt{\frac{\sqrt{5}-1}{\sqrt{5}+1}} + \sqrt{\frac{3-\sqrt{5}}{3+\sqrt{5}}}$$

$$a = \sqrt{\frac{(4-\sqrt{7})(4-\sqrt{7})}{(4+\sqrt{7})(4-\sqrt{7})}} + \sqrt{\frac{(\sqrt{7}-2)(\sqrt{7}-2)}{3(\sqrt{7}+2)(\sqrt{7}-2)}}$$

$$a = \sqrt{\frac{(4-\sqrt{7})^2}{16-7}} + \sqrt{\frac{(\sqrt{7}-2)^2}{3(7-4)}}$$

$$a = \sqrt{\frac{(4-\sqrt{7})^2}{9}} + \sqrt{\frac{(\sqrt{7}-2)^2}{3(7-4)}}$$

$$a = \sqrt{\frac{(4-\sqrt{7})^2}{9}} + \sqrt{\frac{(\sqrt{7}-2)^2}{9}}$$

$$a = \frac{|4-\sqrt{7}|}{3} + \frac{|\sqrt{7}-2|}{3} =$$

$$= \frac{4-\sqrt{7}+\sqrt{7}+2}{3} = \frac{2}{3}$$

$$b = \sqrt{\frac{(\sqrt{5}-1)(\sqrt{5}-1)}{(\sqrt{5}+1)(\sqrt{5}-1)}} + \sqrt{\frac{(3-\sqrt{5})(3-\sqrt{5})}{(3+\sqrt{5})(3-\sqrt{5})}}$$

$$b = \sqrt{\frac{(\sqrt{5}-1)^2}{5-1}} + \sqrt{\frac{(3-\sqrt{5})^2}{9-5}}$$

$$b = \frac{|\sqrt{5}-1|}{2} + \frac{|3-\sqrt{5}|}{2}$$

$$b = \frac{\sqrt{5}-1+3-\sqrt{5}}{2} = \frac{2}{2} = 1$$

$$a = \frac{2}{3} \Rightarrow A < B$$

$$b = 1$$

6. Sa se determine multimea :

$$A = \{n \in \mathbb{Z} \mid \frac{\sqrt{7+4\sqrt{3}} - \sqrt{5-2\sqrt{6}} + \sqrt{11-6\sqrt{2}}}{3n+1} \in \mathbb{Z}\}$$

$$\sqrt{a \pm \sqrt{b}} = \sqrt{\frac{a + \sqrt{a^2 - b}}{2}} \pm \sqrt{\frac{a - \sqrt{a^2 - b}}{2}}$$

$$\sqrt{4+4\sqrt{3}} = \sqrt{7+\sqrt{48}} =$$

$$= \sqrt{\frac{7+\sqrt{49-48}}{2}} + \sqrt{\frac{7-\sqrt{49-48}}{2}} = \sqrt{\frac{7+1}{2}} + \sqrt{\frac{7-1}{2}} = 2 + \sqrt{3}$$

$$\begin{aligned} \sqrt{5-2\sqrt{6}} &= \sqrt{5-\sqrt{24}} = \sqrt{\frac{5+\sqrt{25-24}}{2}} - \sqrt{\frac{5-\sqrt{25-24}}{2}} = \sqrt{\frac{5+1}{2}} - \sqrt{\frac{5-1}{2}} = \\ &= \sqrt{3} - \sqrt{2} \end{aligned}$$

$$\sqrt{11-6\sqrt{2}} = \sqrt{11-\sqrt{72}} = \sqrt{\frac{11+\sqrt{121-72}}{2}} - \sqrt{\frac{11-\sqrt{121-72}}{2}} = \sqrt{\frac{11+7}{2}} - \sqrt{\frac{11-7}{2}} = 3 - \sqrt{2}$$

$$\frac{2 + \sqrt{3} - (\sqrt{3} - \sqrt{2}) + 3 - \sqrt{2}}{3n+1} \in \mathbb{Z}$$

$$\frac{2 + \sqrt{3} - \sqrt{3} + \sqrt{2} + 3 - \sqrt{2}}{3n+1} \in \mathbb{Z}$$

$$\frac{5}{3n+1} \in \mathbb{Z}$$

$$3n + 1/5$$

$$D5 = \{\pm 1, \pm 5\}$$

$$3n + 1 = 1 \Rightarrow 3n = 0 \Rightarrow n = \frac{0}{3} = 0 \in \mathbb{Z}$$

$$3n + 1 = -1 \Rightarrow 3n = -2 \Rightarrow n = \frac{-2}{3} \notin \mathbb{Z}$$

$$3n + 1 = 5 \Rightarrow 3n = 4 \Rightarrow n = \frac{4}{3} \notin \mathbb{Z}$$

$$3n + 1 = -5 \Rightarrow 3n = -6 \Rightarrow n = \frac{-6}{3}$$

$$A = \{-2 : 0\}$$

7. Sa se determine $m \in \mathbb{R}$ stiind ca $(\sqrt{2+\sqrt{3}} + \sqrt{2-\sqrt{3}})^2 - m\sqrt{2} = 0$

Rezolvare:

$$(\sqrt{2+\sqrt{3}} + \sqrt{2-\sqrt{3}})^2 = 2 + \sqrt{3} + 2\sqrt{2+\sqrt{3}} * \sqrt{2-\sqrt{3}} + 2 - \sqrt{3} = 4 + 2\sqrt{4-3} = 4 + 2 = 6$$

$$6 - m\sqrt{2} = 0$$

$$6 = m\sqrt{2}$$

$$m = \frac{\sqrt{2}}{\sqrt{2}} \frac{6}{\sqrt{2}} = \frac{6\sqrt{2}}{2} = 3\sqrt{2}$$

Tema pentru acasa :

1. Sa se afle x din egalitatea :

$$\frac{\sqrt{3+\sqrt{5}}+\sqrt{2-\sqrt{3}}}{\sqrt{5+\sqrt{3}}} = \frac{x}{\sqrt{2}}$$

2) Sa se efectueze :

$$(\sqrt{5}-\sqrt{7})(\sqrt{5}+\sqrt{7}) - \frac{(2\sqrt{5}-5\sqrt{2}) * \sqrt{10}}{\sqrt{5}-\sqrt{2}}$$

3) Se considera $a \in R$ $0 < a < 1$.

$$\text{Calculati } E(a) = \frac{\sqrt{\frac{a^2+2a+1}{a}} + \sqrt{a + \frac{1}{a} - 2}}{\sqrt{a + \frac{1}{a} + 2} - \sqrt{\frac{a^2-2a+1}{a}}}$$

4) Sa se arate ca numarul $a = \sqrt{3-2\sqrt{2}} - \sqrt{3+2\sqrt{2}} + 2$ este numar natural.

5) Sa se calculeze x^4 , unde:

$$x = \sqrt{\sqrt{2}(\sqrt{2}-\sqrt{3})} - \sqrt{3}(\sqrt{2}-\sqrt{3}) + \sqrt{\sqrt{2}(\sqrt{2}+\sqrt{3})} + \sqrt{3}(\sqrt{2}+\sqrt{3})$$